EECS 391
BRANICKY

ANOTHER EXAMPLE OF EXACT INFERENCE

$$P(\neg l \mid \neg m) = P(\neg l, \neg m) / P(\neg m)$$
$$= \alpha$$

$$P(\neg l, \neg m) = \sum_{B=b,\neg b} \sum_{G=g,\neg g} P(G, B, \neg m, \neg l)$$

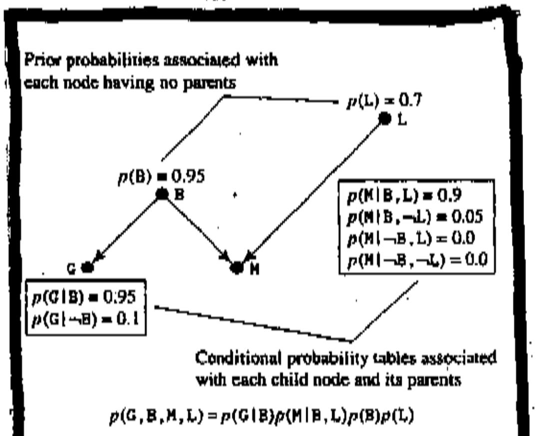


Figure 19.2
A Bayes Network [NILSSON]

$$= \sum_{B} \sum_{G}$$

$$= \underbrace{P(\neg l)}_{0.3} \Big[\underbrace{\sum_{B} P(\neg m \mid B, \neg l) P(B)}_{\overset{\shortparallel}{0.9525}} \underbrace{\sum_{G} P(G \mid B)}_{1} \Big]$$

$$= 0.28575$$

$$\therefore \; P(\neg l \mid \neg m) = \alpha$$

LIKEWISE, $P(l \mid \neg m) = \alpha \, P(l, \neg m) = \alpha \, 0.1015$

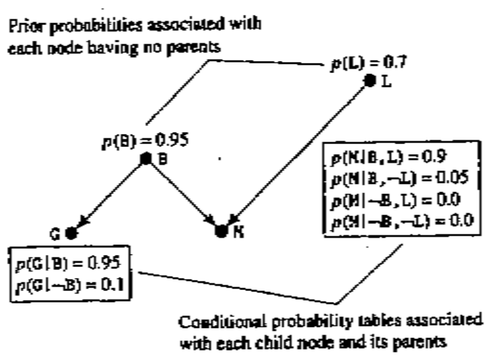
$$\boxed{\begin{aligned} P(l, \neg m) &= \underbrace{P(l)}_{0.7} \Big[\underbrace{\sum_{B} P(\neg m \mid B, l) P(B)}_{} \underbrace{\sum_{G} P(G \mid B)}_{1} \Big] = 0.1015 \\ & P(\neg m \mid b, l) P(b) + \\ & P(\neg m \mid \neg b, l) P(\neg b) \\ & = (0.1)(0.95) + (1)(0.05) = 0.145 \end{aligned}}$$

NORMALIZING, $P(\neg l \mid \neg m) = \quad = \boxed{0.7379}$

```
% a monte carlo trial for computing p(~L|~M)

% before running this, set: query = 0; evidence=0; pall=[];

trials=10000;
for i=1:trials,
  r=rand(1,4);
  B=0; L=0; G=0; M=0;
  if r(1)<0.95,
    B=1;
  end;
  if r(2)<0.7,
    L=1;
  end;
  if (B)
    if(r(3)<0.95)
      G=1;
    end;
    if (L)
      if(r(4)<0.9)
        M=1;
      end;
    else
      if(r(4)<0.05)
        M=1;
      end;
    end;
  else
    if(r(3)<0.1)
      G=1;
    end;
    if (L)
      if(r(4)<0.0)
        M=1;
      end;
    else
      if(r(4)<0.0)
        M=1;
      end;
    end;
  end;
  if (1-M)
    evidence=evidence+1;
    if (1-L)
      query=query+1;
    end;
    p=query/evidence;
    pall=[pall p];
  end;
end;
```



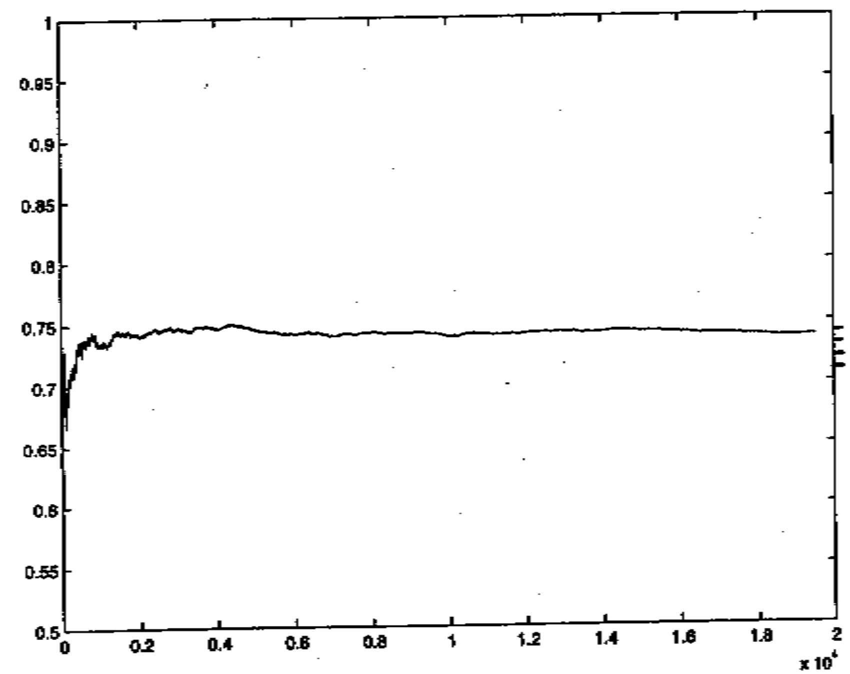
$p(G,B,M,L) = p(G|B)p(M|B,L)p(B)p(L)$

Figure 19.2

A Bayes Network [NILSSON]

APPROXIMATE INFERENCE

– MONTE CARLO SIM. USING REJECTION SAMPLING



Example 5.4. Let x_1 be the speed of a car, x_2 be the acceleration, and y be the force applied to the accelerator. Consider the following fuzzy IF-THEN rule:

$$IF\ x_1\ is\ slow\ and\ x_2\ is\ small,\ THEN\ y\ is\ large \tag{5.33}$$

where "slow" is the fuzzy set defined in Fig. 5.1, that is,

$$\mu_{slow}(x_1) = \begin{cases} 1 & if\ \ x_1 \le 35 \\ \frac{55-x_1}{20} & if\ \ 35 < x_1 \le 55 \\ 0 & if\ \ x_1 > 55 \end{cases} \tag{5.34}$$

"small" is a fuzzy set in the domain of acceleration with the membership function

$$\mu_{small}(x_2) = \begin{cases} \frac{10-x_2}{10} & if\ \ 0 \le x_2 \le 10 \\ 0 & if\ \ x_2 > 10 \end{cases} \tag{5.35}$$

and "large" is a fuzzy set in the domain of force applied to the accelerator with the membership function

$$\mu_{large}(y) = \begin{cases} 0 & if\ \ y \le 1 \\ y-1 & if\ \ 1 \le y \le 2 \\ 1 & if\ \ y > 2 \end{cases} \tag{5.36}$$

Let the domains of x_1, x_2 and y be $U_1 = [0,100], U_2 = [0,30]$, and $V = [0,3]$, respectively. If we use algebraic product for the t-norm in (5.16), then the fuzzy proposition

$$FP_1 = x_1\ is\ slow\ and\ x_2\ is\ small \tag{5.37}$$

is a fuzzy relation in $U_1 \times U_2$ with the membership function

$$\mu_{FP_1}(x_1,x_2) = \mu_{slow}(x_1)\mu_{small}(x_2) = \begin{cases} 0 & if\ x_1 \ge 55\ or\ x_2 > 10 \\ \frac{10-x_2}{10} & if\ x_1 \le 35\ and\ x_2 \le 10 \\ \frac{(55-x_1)(10-x_2)}{200} & if\ 35 < x_1 \le 55\ and\ x_2 \le 10 \end{cases} \tag{5.38}$$

Fig. 5.3 illustrates how to compute $\mu_{FP_1}(x_1,x_2)$.

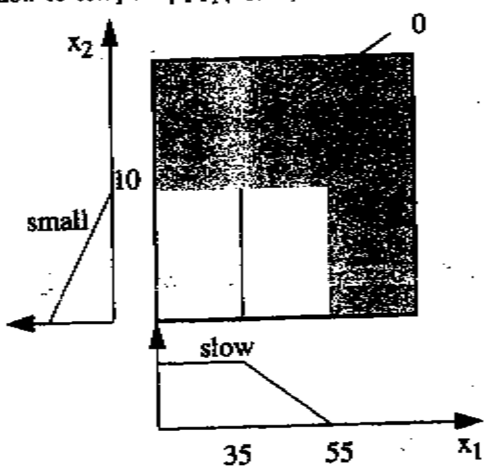


Figure 5.3. Illustration for how to compute $\mu_{slow}(x_1)\mu_{small}(x_2)$ in Example 5.4.

[SOURCE: Li-Xin Wang, A Course in Fuzzy Systems + Control]

FUZZY IF-THEN RULES

If we use the Dienes-Rescher implication (5.23), then the fuzzy IF-THEN rule (5.33) is interpreted as a fuzzy relation $Q_D(x_1,x_2,y)$ in $U_1 \times U_2 \times V$ with the membership function

$$\mu_{Q_D}(x_1,x_2,y) = max[1-\mu_{FP_1}(x_1,x_2), \mu_{large}(y)] \tag{5.39}$$

From (5.38) we have

$$1-\mu_{FP_1}(x_1,x_2) = \begin{cases} 1 & if\ \ x_1 \ge 55\ or\ x_2 > 10 \\ x_2/10 & if\ \ x_1 \le 35\ and\ x_2 \le 10 \\ 1-\frac{(55-x_1)(10-x_2)}{200} & if\ \ 35 < x_1 \le 55\ and\ x_2 \le 10 \end{cases} \tag{5.40}$$

To help us combining $1-\mu_{FP_1}(x_1,x_2)$ of (5.40) with $\mu_{large}(y)$ of (5.36) using the *max* operator, we illustrate in Fig. 5.4 the division of the domains of $1-\mu_{FP_1}(x_1,x_2)$ and $\mu_{large}(y)$ and their combinations. From Fig. 5.4, we obtain

$$\mu_{Q_D}(x_1,x_2,y) = \begin{cases} 1 & if\ x_1 \ge 55\ or\ x_2 > 10\ or\ y > 2 \\ x_2/10 & if\ x_1 \le 35\ and\ x_2 \le 10\ and\ y \le 1 \\ 1-\frac{(55-x_1)(10-x_2)}{200} & if\ 35 < x_1 \le 55\ and\ x_2 \le 10\ and\ y \le 1 \\ max[y-1, x_2/10] & if\ x_1 \le 35\ and\ x_2 \le 10\ and\ 1 < y \le 2 \\ max[y-1, 1-\frac{(55-x_1)(10-x_2)}{200}] & if\ 35 < x_1 \le 55\ and\ x_2 \le 10\ and\ 1 < y \le 2 \end{cases} \tag{5.41}$$

For Lukasiewicz, Zadeh and Mamdani implications, we can use the same procedure to determine the membership functions. □

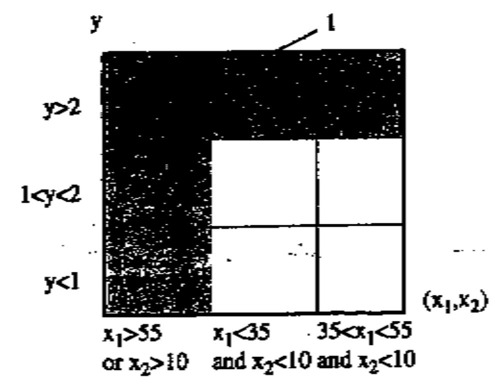


Figure 5.4. Division of the domains of $1-\mu_{FP_1}(x_1,x_2)$ and $\mu_{large}(y)$ and their combinations for Example 5.4.

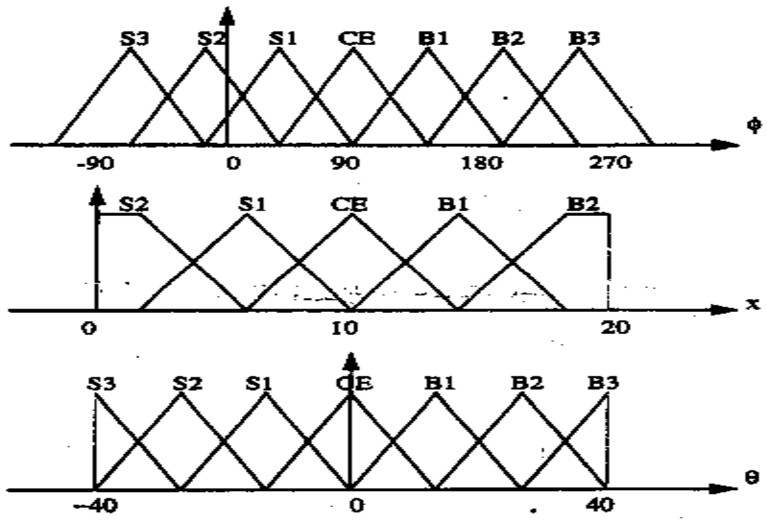
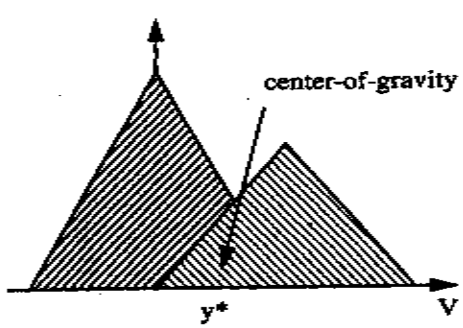


Figure 12.5. Membership functions for the truck backer-upper control problem.

ϕ \ x	S2	S1	CE	B1	B2
S3	S2	S3			
S2	S2	S3	S3	S3	
S1	B1	S1	S2	S3	S2
CE	B2	B2	CE	S2	S2
B1	B2	B3	B2	B1	S1
B2		B3	B3	B3	B2
B3				B3	B2

Figure 12.6. The final fuzzy rule base for the truck backer-upper control problem.



CENTER-OF-GRAVITY DEFUZZIFICATION

$$y^* = \frac{\int_V y\mu_{B'}(y)dy}{\int_V \mu_{B'}(y)dy}$$

FUZZY CONTROL OF TRUCK BACKER-UPPER

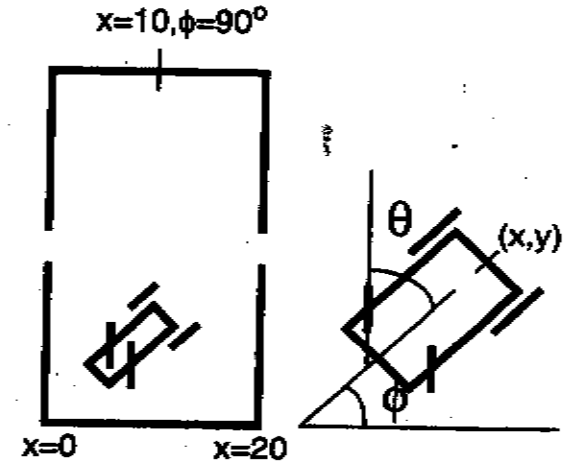


Figure 12.4. The simulated truck and loading zone.

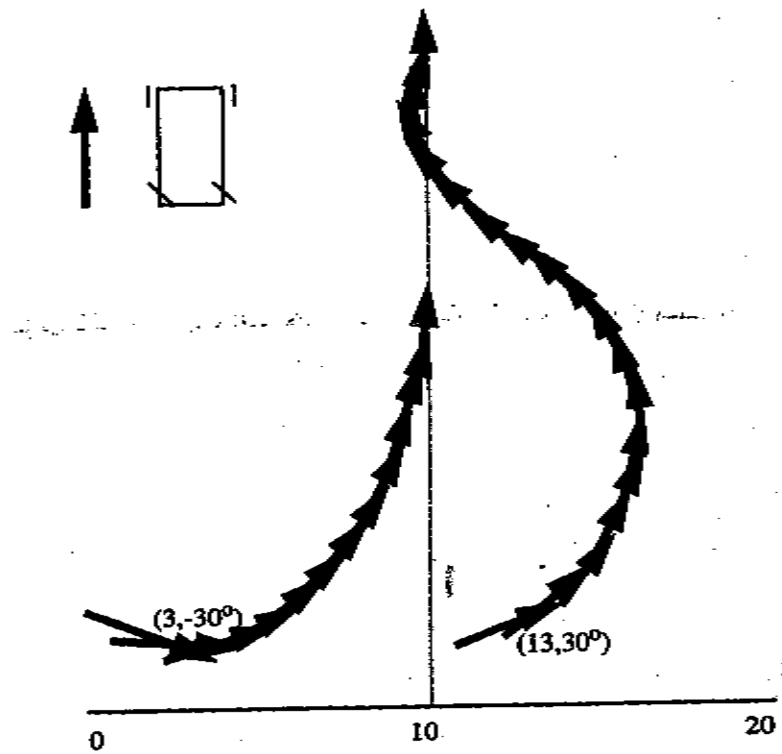


Figure 12.7. Truck trajectories using the fuzzy controller.

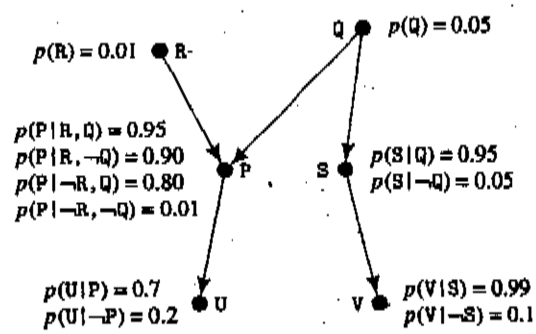


Figure 19.5

A Small Polytree

The top-down algorithm successively calculates

$$p(U|Q) = \sum_{P} p(U|P)p(P|Q)$$

$$\begin{aligned} p(P|Q) &= \sum_{R} p(P|R, Q)p(R) \\ &= p(P|R, Q)p(R) + p(P|\neg R, Q)p(\neg R) \\ &= 0.95 \times 0.01 + 0.8 \times 0.99 = 0.80, \text{ thus } \ldots \end{aligned}$$

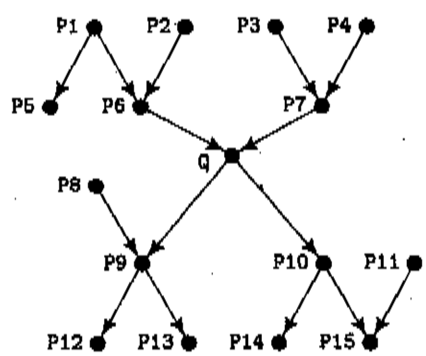
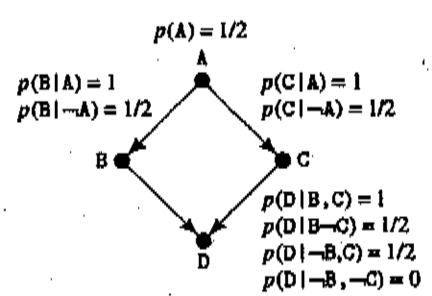


Figure 19.4

A Typical Polytree

19.3 An admissions committee for a college is trying to determine the probability that an admitted candidate is really qualified. The relevant probabilities are given in the Bayes network shown here. Calculate $p(A|D)$.



NOT A POLYTREE

A = applicant is qualified
B = applicant has high grade point average
C = applicant has excellent recommendations
D = applicant is admitted

$p(A|D) = ?$

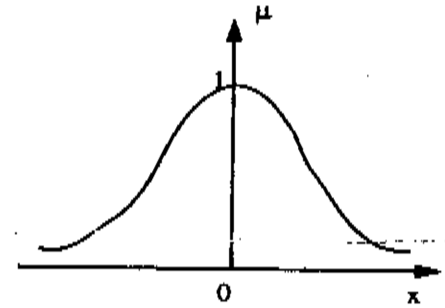


Figure 2.3. A possible membership function to characterize "numbers close to zero."

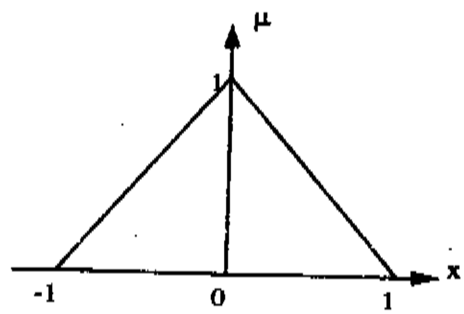


Figure 2.4. Another possible membership function to characterise "numbers close to zero."

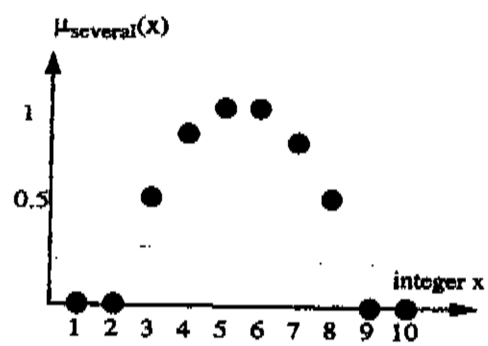


Figure 2.6. Membership function for fuzzy set "several."

[WANG]