GAMES AS TABLES

There is another way of looking at games, one far more useful in game theory. A game is equivalent to a table of possible outcomes.

As we have shown, the number of possible games of chess is astronomically large but finite nevertheless. It follows that the number of chess strategies is finite, too. I have already used the word "strategy" several times; now is the time to define it. In game theory, strategy is an important idea, and it has a more precise meaning than it usually does. When chess players talk of a strategy, they mean something like "open with the king's Indian Defense and play aggressively." In game theory, a strategy is a much more specific plan. It is a *complete* description of a particular way to play a game, no matter what the other player(s) does and no matter how long the game lasts. A strategy must prescribe actions so thoroughly that you never have to make a decision in following it.

An example of a true strategy for first player in ticktacktoe would be:

> Put X in the center square. O can respond two ways:
>
> **1** If O goes in a noncorner square, put X in a corner cell adjacent to the O. This gives you two-in-a-row. If O fails to block on the next move, make three-in-a-row for a *win*. If O blocks, put X in the empty corner cell that is not adjacent to the first (noncorner) O. This gives you two-in-a-row two ways. No matter what O does on the next move, you can make three-in-a-row after that and *win*.
>
> **2** If instead O's first move is a corner cell, put X in one of the adjacent noncorner cells. This gives you two-in-a-row. If O fails to block on the next move, make three-in-a-row for a *win*. If O blocks, put X in the corner cell that is adjacent to the second O and on the same side of the grid as the first O. This gives you two-in-a-row. If O fails to block on the next move, make three-in-a-row for a *win*. If O blocks, put X in the empty cell adjacent to the third O. This gives you two-in-a-row. If O fails to block on the next move, make three-in-a-row for a *win*. If O blocks, fill in the remaining cell for a *tie*.

This shows how complicated a strategy can be, even for a very simple game. A true strategy for chess would be so huge that it could never be written down. There is not enough paper and ink on earth to list all the possibilities; there is not enough computer memory to loop through them all. This is one reason why computers still aren't unbeatable at chess.

Overwhelming as this practical difficulty is, it didn't bother von Neumann, and it needn't bother us. In fact, since we're fantasizing, we might as well go a step further. Not only could a perfectly rational

being conceive of a strategy in full detail; he could—given no limits on memory or computing power whatsoever—anticipate every possible chess strategy and decide in advance what to do even before moving the first piece.

Suppose you had a numbered list of all possible strategies for chess. Then your choice of strategy is tantamount to selecting a number from 1 to n, where n is the (very, very large) number of possible strategies. Your opponent could choose a strategy from his list of possibilities (from 1 to m, say). Once these two strategies were chosen, the resulting game would be completely specified. By applying the two strategies you could move the pieces and bring the game to its preordained conclusion. Openings, captures, "surprise moves," and endgame would all be implicit in the choice of strategies.

To take this pipe dream to its conclusion, we can imagine that, given enough time, you could play out every possible pairing of strategies to see the outcome. The results could be tabulated in a rectangular table. The real table would span the galaxies, so we'll print an abbreviated version here!

		Black's Strategies				
		1	2	3	. . .	m
White's Strategies	1	White checkmate in 37 moves	Draw in 102 moves	Black resigns in 63 moves		Black checkmate in 42 moves
	2	White checkmate in 45 moves	Black checkmate in 17 moves	White checkmate in 54 moves		White checkmate in 82 moves
	3	White checkmate in 43 moves	White checkmate in 108 moves	Draw in 1,801 moves		Black checkmate in 32 moves
	. . .					
	n	Draw in 204 moves	White checkmate in 77 moves	White checkmate in 24 moves		Draw in 842 moves

Once you had this table, you wouldn't have to bother with the chessboard anymore. A "game" of chess would amount to the two players choosing their strategies simultaneously and looking up the result in the table.[2] To find out who wins, you'd look in the cell at the intersection of the row corresponding to White's strategy and the column of Black's strategy. Should White choose to use strategy number 2 on his list, and Black choose to use his strategy number 3, the inevitable outcome would be a checkmate for White in 54 moves.

This isn't the way real people play real games. To detail every possible contingency beforehand would be the antithesis of the word "play." No matter. This idea of representing games as tables of outcomes turns out to be very useful. Every possible sequence of play for any two-person game can be represented as a cell in a similar type of table. The table must have as many rows as one player has strategies and a column for each of the other player's strategies. A game reduced to a table like this is called the "normalized form" of the game.

The trick is deciding *which* strategy to choose. The table gets all the facts out in the open, but this isn't always enough. The arrangement of outcomes in the table can be capricious. Neither player gets to choose the outcome he wants, only the row or column it appears in. The other player's choice makes an equal contribution.

Look at the imaginary table for chess. Is strategy number 1 a good choice for White? It's tough to say. If Black chooses strategy number 1, it's good because that leads to a win for White. But with other choices for Black, the result can be a draw or a loss for White.

White really wants to determine which strategy Black is going to choose. Then all White has to do is make sure he picks one of his own strategies that will lead to a win when paired with the Black strategy.

Unfortunately, Black wants to do the same thing. Black wants to psych out White, and choose his strategy accordingly for a Black victory. Of course, White knows this, and tries to predict what Black will do based on what he thinks White will do . . .

Borel and von Neumann realized that this sort of deliberation puts games beyond the scope of probability theory. The players would be wrong indeed to assume that their opponent's choice is determined by

2. Why "simultaneously"? Doesn't Black at least get to see White's first move before deciding on his strategy? No: you're failing to appreciate how comprehensive a strategy must be. The first part of a Black strategy would prescribe a Black opening move for *each* of the twenty possible opening moves by White. Not until each of these twenty contingencies is accounted for do you have a strategy in von Neumann's sense.

"chance." Chance has nothing to do with it. The players can be expected to do their very best to deduce the other's choice and prepare for it. A new theory is called for.