EECS 391
BRANICKY

PROBLEM SET #7

7.1 (R&N 13.6)

(a) $P(toothache) = 0.108 + 0.012 + 0.016 + 0.064 = \boxed{0.2}$

(b) $\mathbf{P}(Cavity) = \langle P(cavity), P(\neg cavity) \rangle = \boxed{\langle 0.2, 0.8 \rangle}$

(c) $$\mathbf{P}(Toothache \mid cavity) = \langle P(toothache \mid cavity), P(\neg toothache \mid cavity) \rangle = \left\langle \frac{0.108 + 0.012}{0.2}, \frac{0.072 + 0.008}{0.2} \right\rangle = \boxed{\langle 0.6, 0.4 \rangle}$$

(d) $\mathbf{P}(Cavity \mid toothache \vee catch) = \boxed{\langle 0.4615, 0.538 \rangle}$

B/C $$P(cavity \mid toothache \vee catch) = \frac{0.108 + 0.012 + 0.072}{(0.108 + 0.012 + 0.072 + 0.016 + 0.064 + 0.144)} = \frac{0.192}{0.192 + 0.224} = \frac{0.192}{0.416}$$

$$P(\neg cavity \mid toothache \vee catch) = \frac{0.224}{0.192 + 0.224} = \frac{0.224}{0.416}$$

NOTES: ① $toothache \vee catch$ IS TRUE IN 6 OF THE 8 ATOMIC EVENTS WITH THESE PROBABILITIES

② COMPUTED A DIFFERENT WAY:
$$\neg(toothache \vee catch) = \underbrace{\neg toothache \wedge \neg catch}_{\substack{\text{PROB. IS} \\ 0.008 + 0.576 \\ = 0.584}}$$

AND $1 - 0.584 = 0.416$ ✓

③ DENOMINATORS ABOVE ARE SUMS OF NUMERATORS (WHICH IS WHY NORMALIZATION WORKS)

7.2 (RBN 13.8)

TEST IS 99% ACCURATE:

$$P(t|d) = 0.99$$
$$P(\neg t|\neg d) = 0.99$$
$$P(d) = 0.0001$$

PATIENT IS INTERESTED IN $P(d|t)$

$$P(d|t) = \frac{P(t|d)\,P(d)}{P(t)}$$

HENCE, IT IS GOOD NEWS THE DISEASE IS RARE, SINCE THIS DIRECTLY DIMINISHES $P(d|t)$. NUMERICALLY,

$$P(d|t) = \frac{P(t|d)\,P(d)}{P(t|d)\,P(d) + P(t|\neg d)\,P(\neg d)}$$
$$= \frac{(0.99)(0.0001)}{(0.99)(0.0001) + (0.01)(0.9999)}$$
$$= \boxed{0.0098}$$

TAKE-HOME: WHEN DISEASE IS MUCH RARER THAN TEST ACCURACY, A POSITIVE TEST RESULT DOES NOT MEAN THE DISEASE IS LIKELY. INDEED, A FALSE POSITIVE READING IS MUCH MORE LIKELY.

7.3 (R&N 13.12)

$$P(M|s) = P(s|M)\,P(M)/P(s)$$
$$P(\neg M|s) = P(s|\neg M)\,P(\neg M)/P(s)$$

BOTH FROM BAYES' RULE

NOTE ALSO THAT $P(M|s) + P(\neg M|s) = 1$

THUS, SETTING $\frac{1}{P(s)} = \alpha$ WE SEE

$$\alpha\, P(s|M)\,P(M) + \alpha\, P(s|\neg M)\,P(\neg M) = 1$$
$$\alpha = \frac{1}{P(s|M)P(M) + P(s|\neg M)P(\neg M)} = \frac{1}{P(s)}$$

THEREFORE, NORMALIZATION ALLOWS US TO COMPUTE $P(M|s)$ AND $P(\neg M|s)$ WITHOUT KNOWING $P(s)$....

GIVEN: $P(m) = 1/50000$, $P(s|m) = 0.5$, $P(s|\neg m) = 0.05$

UNNORMALIZED:
$$P(M|s) = \alpha \langle P(s|m)P(m),\ P(s|\neg m)P(\neg m)\rangle$$
$$= \alpha \left\langle 0.5/50000,\ (0.05)\left(1 - \tfrac{1}{50000}\right)\right\rangle$$
$$= \alpha \langle 0.00001,\ 0.049999\rangle$$

NORMALIZED: $\alpha = \frac{1}{0.00001 + 0.049999} = \frac{1}{0.050009}$

$$P(M|s) = \left\langle \frac{0.00001}{0.050009},\ \frac{0.049999}{0.050009}\right\rangle$$
$$\doteq \boxed{\langle 0.0002,\ 0.9998\rangle}$$

7.4 (R&N 13.15)

GIVEN INFO. ON RELIABILITY OF ID:

$$P(LB \mid B) = 0.75$$
$$P(\neg LB \mid \neg B) = 0.75$$

WHERE

$LB \equiv$ LOOKED BLUE
$B \equiv$ WAS BLUE

$$P(Color \mid LB) = \langle P(B \mid LB),\ P(\neg B \mid LB) \rangle$$
$$= \alpha \langle P(LB \mid B)\, P(B),\ P(LB \mid \neg B)\, P(\neg B) \rangle$$

$\therefore$ IT IS $\boxed{\text{NOT POSSIBLE}}$ TO CALCULATE THE MOST LIKELY COLOR FOR THE TAXI WITHOUT MORE INFORMATION.

FOR EXAMPLE, IF ALL ARE BLUE, $P(B \mid LB) = 1$ WHILE IF ALL ARE GREEN, $P(B \mid LB) = 0$.

GIVEN, 9 OUT OF 10 ARE GREEN $\Rightarrow P(B) = 0.1$

$$P(Color \mid LB) = \alpha \langle (0.75)(0.1),\ (0.25)(0.9) \rangle$$
$$= \alpha \langle 0.075,\ 0.225 \rangle$$
$$= \frac{1}{0.075 + 0.225} \langle 0.075,\ 0.225 \rangle$$
$$= \frac{1}{0.3} \langle 0.075,\ 0.225 \rangle$$
$$= \boxed{\langle 0.25,\ 0.75 \rangle}$$

THUS, EVEN THOUGH IT LB, IT IS MORE LIKELY TO BE $\neg B$.

7.5 (R&N 13.19)

N/5 PITS AMONG N SQUARES OTHER THAN [1,1].

IN 4×4 WORLD, $N = 16 - 1 = 15 \Rightarrow 3$ PITS

THE VARIABLES $P_{i,j}$ AND $P_{k,l}$ ARE NOW $\boxed{\text{NOT INDEPENDENT}}$ BECAUSE IF $P_{i,j} = true$, IT IS LESS LIKELY THAT $P_{k,l} = true$ (FEWER MINES TO SPREAD AROUND).

THE JOINT DISTRIBUTION ON $P_{1,2}, \ldots, P_{4,4}$ IS

$$\boxed{P(P_{1,2}, \ldots, P_{4,4}) = \begin{cases} 1/\binom{15}{3} = \frac{1}{455}, & \text{IF EXACTLY 3 OF THE } P_{i,j} \text{ ARE TRUE;} \\ 0, & \text{OTHERWISE} \end{cases}}$$

TO RE-DO THE CALCULATIONS FOR $P_{1,3}$ AND $P_{2,2}$ WE MUST CONSIDER THE NUMBER OF MODELS CONSISTENT WITH THE DATA AND THE NUMBER OF THESE IN WHICH EACH OF $P_{1,3}$ AND $P_{2,2}$ ARE TRUE OR FALSE.

LOOKING AT FIGURE 13.7, PAGE 485 OF R&N, THERE ARE

$$\binom{10}{0}, \binom{10}{1}, \binom{10}{1}, \binom{10}{1}, \binom{10}{2},$$
$$= 1, \; 10, \; 10, \; 10, \; 45, \leftarrow 76 \text{ TOTAL}$$

MODELS CONSISTENT WITH EACH PICTURE, RESP.

$$\therefore \quad P(P_{1,3}) = \left\langle \frac{1+10+10}{76}, \frac{10+45}{76} \right\rangle = \boxed{\left\langle \frac{21}{76}, \frac{55}{76} \right\rangle \approx \langle .276, .724 \rangle}$$
$$P(P_{2,2}) = \left\langle \frac{1+10+10+45}{76}, \frac{10}{76} \right\rangle = \boxed{\left\langle \frac{66}{76}, \frac{10}{76} \right\rangle \approx \langle .868, .132 \rangle}$$