Remington Davenport
EECS 491 PS#8
3-28-07

20

Problem G8.1 : 14.11 out of R&N

a. The Markov chain has 4 states, since there are two variables whose values are not specified (Rain & Cloudy) and each can take two possible values.

b. Find the distributions of Rain and Cloudy conditioned under all possible values of their Markov blanket (that agree w/ the evidence). Cloudy's blanket is Rain & Sprinkler = true :

$$\underline{P}(C \mid r,s) = \alpha \underline{P}(C,r,s)$$
$$= \alpha \langle P(c,r,s), P(\neg c,r,s) \rangle$$
$$= \alpha \langle P(c)P(r|c)P(s|c), P(\neg c)P(r|\neg c)P(s|\neg c) \rangle$$
$$= \alpha \langle (0.5)(0.8)(0.1), (0.5)(0.2)(0.5) \rangle$$
$$= \alpha \langle 0.04, 0.05 \rangle = \langle 4/9, 5/9 \rangle$$

$$\underline{P}(C \mid \neg r,s) = \alpha \langle P(c)P(\neg r|c)P(s|c), P(\neg c)P(\neg r|\neg c)P(s|\neg c) \rangle$$
$$= \alpha \langle (0.5)(0.2)(0.1), (0.5)(0.8)(0.5) \rangle$$
$$= \alpha \langle 0.01, 0.2 \rangle = \langle 1/21, 20/21 \rangle$$

Rain's Markov blanket has Cloudy, Wet Grass = true, & Sprinkler = true:

$$\underline{P}(R \mid c,w,s) = \alpha \underline{P}(R,c,w,s)$$
$$= \alpha \langle P(r,c,w,s), P(\neg r,c,w,s) \rangle$$
$$= \alpha \langle P(c)P(r|c)P(s|c)P(w|r,s), P(c)P(\neg r|c)P(s|c)P(w|\neg r,s) \rangle$$
$$= \alpha \langle (0.5)(0.8)(0.1)(0.99), (0.5)(0.2)(0.1)(0.9) \rangle$$
$$= \alpha \langle 0.0396, 0.009 \rangle = \langle 22/27, 5/27 \rangle$$

$$\underline{P}(R \mid \neg c,w,s) = \alpha \langle P(\neg c)P(r|\neg c)P(s|\neg c)P(w|r,s), P(\neg c)P(\neg r|\neg c)P(s|\neg c)P(w|\neg r,s) \rangle$$
$$= \alpha \langle (0.5)(0.2)(0.5)(0.99), (0.5)(0.8)(0.5)(0.9) \rangle$$
$$= \alpha \langle 0.0495, 0.18 \rangle = \langle 11/51, 40/51 \rangle$$

We can then use these to fill the transition matrix :

Four possible states: (r,c), $(r,\neg c)$, $(\neg r,c)$, $(\neg r,\neg c)$

$\underline{Q}$:

$Y \backslash Y'$	(r,c)	$(r,\neg c)$	$(\neg r,c)$	$(\neg r,\neg c)$
(r,c)	$17/27$	$5/18$	$5/54$	0
$(r,\neg c)$	$2/9$	$59/153$	0	$20/51$
$(\neg r,c)$	$11/27$	0	$22/189$	$10/21$
$(\neg r,\neg c)$	0	$11/102$	$1/42$	$310/357$

The anti-diagonal is 0 since the values of both variables cannot change. For entries where 1 value changes, it is 0.5 times P(new value | old values) (the 0.5 comes from the fact that there's a 50% chance that variable is sampled).

$$q((\neg r,c) \to (r,c)) = 0.5\,P(r \mid c,s,w) = (0.5)(22/27) = 11/27$$
$$q((r,c) \to (\neg r,c)) = 0.5\,P(\neg r \mid c,s,w) = (0.5)(5/27) = 5/54$$
$$q((r,c) \to (r,\neg c)) = 0.5\,P(\neg c \mid r,s) = 5/18$$
$$q((r,\neg c) \to (r,c)) = 0.5\,P(c \mid r,s) = 2/9$$
$$q((\neg r,\neg c) \to (r,\neg c)) = 0.5\,P(r \mid \neg c,s,w) = 11/102$$
$$q((\neg r,\neg c) \to (\neg r,c)) = 0.5\,P(c \mid \neg r,s) = 1/42$$
$$q((r,\neg c) \to (\neg r,\neg c)) = 0.5\,P(\neg r \mid \neg c,s,w) = 20/51$$
$$q((\neg r,c) \to (\neg r,\neg c)) = 0.5\,P(\neg c \mid \neg r,s) = 10/21$$

For entries where neither value changes, it's 0.5 times the sum of probabilities of each variable not changing:

$$q((r,c) \to (r,c)) = 0.5\left[P(r \mid c,s,w) + P(c \mid r,s)\right] = 0.5\left[22/27 + 4/9\right] = 17/27$$
$$q((r,\neg c) \to (r,\neg c)) = 0.5\left[P(r \mid \neg c,s,w) + P(\neg c \mid r,s)\right] = 59/153$$
$$q((\neg r,c) \to (\neg r,c)) = 0.5\left[P(\neg r \mid c,s,w) + P(c \mid \neg r,s)\right] = 22/189$$
$$q((\neg r,\neg c) \to (\neg r,\neg c)) = 0.5\left(P(\neg r \mid \neg c,s,w) + P(\neg c \mid \neg r,s)\right] = 310/357$$

c. $\underline{Q}^2$ represents the probability of going from one state to another in two steps.

d. $\underline{Q}^n$ as $n \to \infty$ represents the probability of being in each state after a long time for each of the different starting states.

e. As $n \to \infty$, $\underline{Q}^n$'s values will approach the posterior probabilities, but finding $\underline{Q}^n$ takes a long time so it is not reasonable to generate ($\underline{Q}$ is exponential in size on number of variables, and matrix multiplication is cubic-time).

$O((2^n)^3)$ FOR n VARIABLES