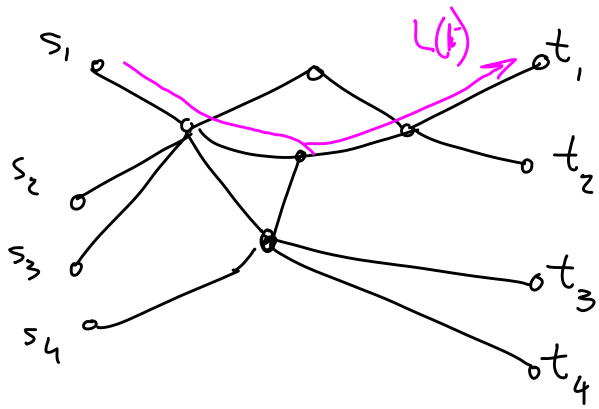
Lecture 13

Note Title 2/11/2005

Optimization-based congestion control



$U_i(x_i)$ = utility of flow i (as a function of the throughput x_i).

Eg $U_i(x_i) = \log x_i$

$$U^* = \max \sum_i U_i(x_i)$$

$$\text{s.t.} \quad \sum_{i \in S(\ell)} x_i \le c_\ell \qquad \forall \ell \in E$$

$$\cancel{x_i \ge 0} \qquad 0 < m \le x_i \le M$$

$$L(\underline{x}, \underline{p}) = \sum_i U_i(x_i) - \sum_{\ell \in E} p_\ell \left(\sum_{i \in S(\ell)} x_i - c_\ell \right)$$

$$\forall \underline{p} \ge 0 \qquad \max_{\underline{x}} L(\underline{x}, \underline{p}) \ge U^*$$

$$\exists \underline{p}^* \qquad \max_{\underline{x}} L(\underline{x}, \underline{p}) = U^*$$

$$L(\underline{x}, \underline{p}) = \sum_i \left[U_i(x_i) - x_i \sum_{\ell \in L(i)} p_\ell \right] + \sum_\ell p_\ell c_\ell$$

$$\Rightarrow \max \; U_i(x_i) - x_i \underbrace{\sum_{\ell \in L(i)} p_\ell}_{p_i} \qquad \forall i$$

$$\max \underbrace{U_i(x_i)}_{x_i = \infty} - x_i p_i$$

$$\boxed{x_i = U'^{-1}(p_i)}$$

Eg $U_i = \log$

$p_i = 2$

$$\max \log x - 2x$$

$$\frac{1}{x} - 2 = 0 \qquad x = \frac{1}{2}$$

$$U'(x) = p \qquad x = U'^{-1}(p)$$

Complementary slackness

$$p_\ell = 0 \Leftarrow \sum_{i \in S(\ell)} x_i < c_\ell$$

Idea: $p_\ell = \left[\sum_{i \in S(\ell)} x_i - c_\ell\right]^+$

$$p_\ell \leftarrow \left[p_\ell + \gamma\left(\sum_{i \in S(\ell)} x_i - c_\ell\right)\right]^+$$

$$q_\ell = \varphi^{p_\ell} \qquad (\varphi < 1) \qquad p_\ell \geq 0 \Rightarrow q_\ell \leq 1$$

Mark packet w/ probability q_ℓ

$$\Pr[\text{marked packet}] = q_1 + (1-q_1)q_2 + \dots + \dots$$

$$\underbrace{q_1 q_2 \cdots q_m}_{\varphi^{p_1 + p_2 + \dots + p_m}}$$

Mark packet w/ probability $1 - q_\ell$

$$\Pr[\text{packet marked}] = 1 - (1-q_1)(1-q_2)\cdots(1-q_m) = 1 - \varphi^{p_1+p_2+\cdots+p_m}$$

REM : random exponential marking. (ns)