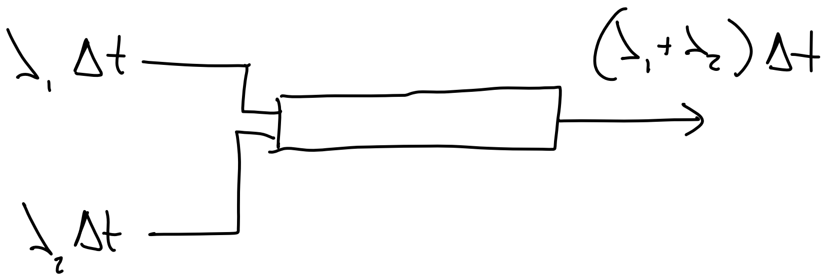
Lecture 21

Note Title 3/2/2005

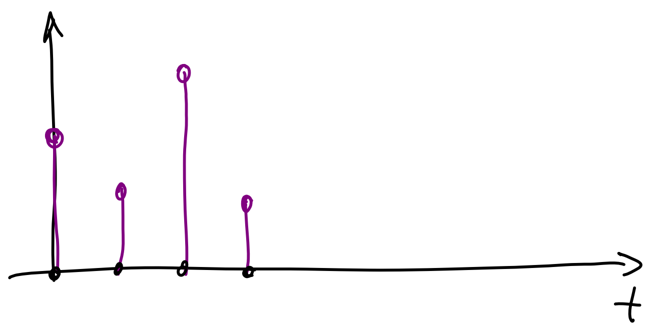
Traffic characterization

$$\Pr[X_t = n] = e^{-\lambda \Delta t} \frac{(\lambda \Delta t)^n}{n!}$$

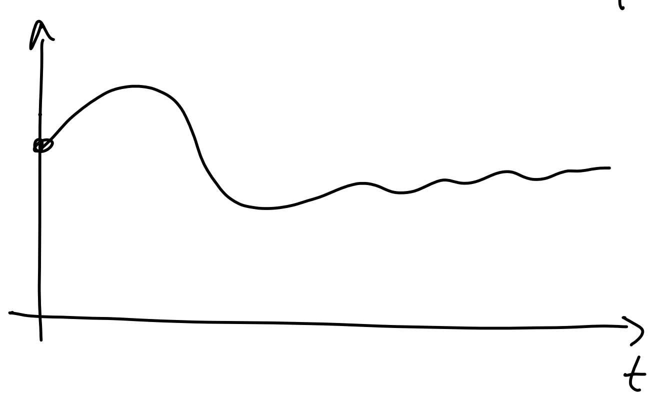


Stochastic process: set of random variables $\{x(t) : t \in T\}$

Eg $T = \mathbb{N}$ (time series)



$T = \mathbb{R}$



$$E[x(t)] = \mu(t)$$

Autocorrelation function $R : T \times T \to \mathbb{R}$

$$R(t_1, t_2) = E[x(t_1) x(t_2)].$$

If $x(t_1), x(t_2)$ were indpt $R(t_1,t_2) = \mu(t_1)\mu(t_2)$

Autocovariance fcn $C(t_1,t_2) = R(t_1,t_2) - \mu(t_1)\mu(t_2)$

Correlation coefficient $\rho(t_1,t_2) = \dfrac{C(t_1,t_2)}{\sigma_1 \sigma_2}$ $\sigma_i^2 = Var[x(t_i)]$

Independent increments

$\forall t_0, t_1, \ldots, t_n : t_0 < t_1 < \ldots < t_n$

$x(t_1) - x(t_0), x(t_2) - x(t_1), \ldots, x(t_n) - x(t_{n-1})$

are independent.

Stationary increments increments are stationary.

Stationary stochastic process (wide sense stationary)

- $E[x(t)] = \mu$
- $R(t, t+\tau) = R(t+\tau, t) = R(\tau) = R(-\tau)$ $(\tau \geq 0)$

Stationary indpt increments $\Rightarrow$ $E[x(t)] = at + b$

if continuous fcn

$Var[x(t) - x(0)] = \sigma^2 t$.

Poisson Counting process $\{N(t), t \geq 0\}$

1. $N(t)$ has stationary indpendent increments
2. $N(0) = 0$

3. $\Pr[N(t_2)-N(t_1)=n] = \frac{[\lambda(t_2-t_1)]^n}{n!} e^{-\lambda(t_2-t_1)}$

Poisson increment process

$$X(t) = \frac{N(t+L)-N(t)}{L}$$

stationary (by property 1).

$$R(\tau) = \begin{cases} \lambda^2 & |\tau|>L \\ \text{- - -} & |\tau|<L \end{cases}$$

$$R(\tau) = E[X(t+\tau)X(t)] = \frac{1}{L^2} E\{[N(t+\tau+L)-N(t+\tau)][N(t+L)-N(t)]\} =$$

indpt increments

$$= \frac{1}{L^2} E[N(t+\tau+L)-N(t+\tau)]E[N(t+L)-N(t)]$$

$$= \ldots = \lambda^2.$$