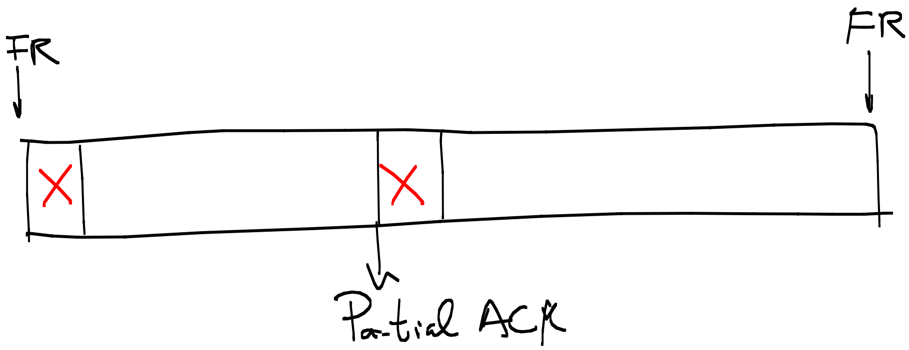
Lecture 22

Note Title 3/13/2005

Midterm distribution

70+	2
56-64	4
40-	1



Traffic characterization

Stochastic process $\{x(t) : t \in T\}$

Autocorrelation function $R(t_1, t_2) = E[x(t_1)x(t_2)]$.

Brownian motion $B(t)$

Def.
1. $\{B(t) : 0 \le t < \infty\}$ has stationary indpt increments
2. $\forall t > 0$ $B(t)$ has normal distribution
3. $\forall t > 0$ $E[B(t)] = 0$
4. $B(0) = 0$

Properties $Var[x(t)] = \sigma^2 t$ (in general) for $B(t)$ $\quad \mathrm{Var}[B(t)] = t$

$$\mathrm{Var}[B(t+s)-B(t)] = \mathrm{Var}[B(s)-B(0)] = \mathrm{Var}[B(s)] = s.$$

$$E[B(t+s)-B(t)] = E[B(t+s)] - E[B(t)] \underset{(3)}{=} 0$$

$B(t) = B(t) - B(0)$ ← normal rnd var

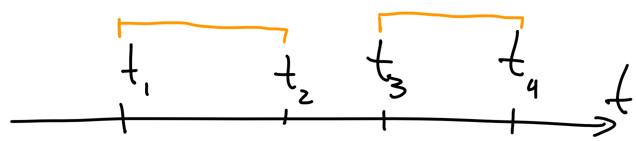
normal rnd var

same distributionally as $B(t+s)-B(s)$ (by stationary increment)

$$\Pr[B(t+\delta)-B(t) \le x] = \frac{1}{\sqrt{2\pi\delta}} \int_{-\infty}^{x} e^{-\frac{y^2}{2\delta}}\, dy \quad \text{(Normal)}$$

$$R(t_1, t_2) = E[B(t_1) B(t_2)]$$

$\forall\, t_1 < t_2 \le t_3 < t_4$



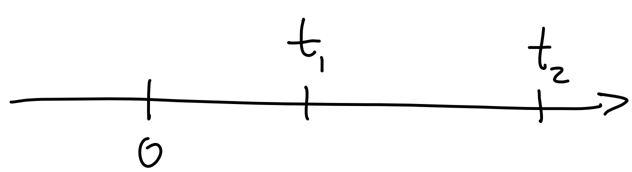
$$E[(B(t_4)-B(t_3))(B(t_2)-B(t_1))] = \quad \text{→ independence}$$

$$E[B(t_4)-B(t_3)] \cdot E[B(t_2)-B(t_1)] = 0$$

$t_1 = 0$

$t_2 = t_3 \rightarrow t_1$

$t_4 \rightarrow t_2$



$$0 = E[(B(t_2)-B(t_1))(B(t_1)-B(0))] =$$

$$= E[(B(t_2)-B(t_1))B(t_1)] =$$

$$= E[B(t_2)B(t_1)] - E[B^2(t_1)] =$$

$$= R(t_1, t_2) \qquad - \quad Var[B(t_1)] =$$

$$= R(t_1, t_2) \qquad - t_1$$

$R(t_1, t_2) = t_1$. $\qquad\qquad t_1 < t_2$

$$Var[X] = E[X^2] - (E[X])^2$$

$$Var[B(t_1)] = E[B^2(t_1)] - 0$$

1

$$\rho(t_1, t_2) = \frac{R(t_1, t_2) - E[B(t_1)]\,E[B(t_2)]}{\sqrt{Var[B(t_1)]\,Var[B(t_2)]}} = \frac{t_1}{\sqrt{t_1 t_2}} = \sqrt{\frac{t_1}{t_2}}$$