Lecture 24

Note Title 3/18/2005

Self-similarity $\quad x(t) \overset{d}{=} \dfrac{x(at)}{a^H} \quad \forall a>0$

$\frac{1}{2} \leq H < 1$

1. $E[x(t)] = \dfrac{E[x(at)]}{a^H}$

2. $Var[x(t)] = \dfrac{Var[x(at)]}{a^{2H}}$

3. $R(t,s) = \dfrac{R(at, as)}{a^{2H}}$

$fBm(H)$ is self-similar

$E[B_H(t)] = 0 \quad \forall t \geq 0$

$Var[B_H(t)] = t^{2H} \qquad Var[B_H(at)] = a^{2H} t^{2H}$

$R(t,s) = \frac{1}{2}\left(t^{2H} + s^{2H} - |t-s|^{2H}\right)$

B_H is self-similar

$A(t)$ = cumulative traffic from initial time (eg $t=0$) to time t

$$A(t) = mt + a B_H(t)$$

Questions:

1. Implications for QoS (queuing delays? BW provisioning? Buffer provisioning?)
2. How does B_H arise in Internet?

Origin of A(t)

Superposition of stochastic processes.

$$\{ W^{(i)}(t) : t \geq 0 \} \qquad 1 \leq i \leq M$$

Their (normalized) superposition

$$W_M^*(Tt) = \int_0^{tT} \sum_{i=1}^{M} W^{(i)}(u)\, du$$

Heavy-tailed

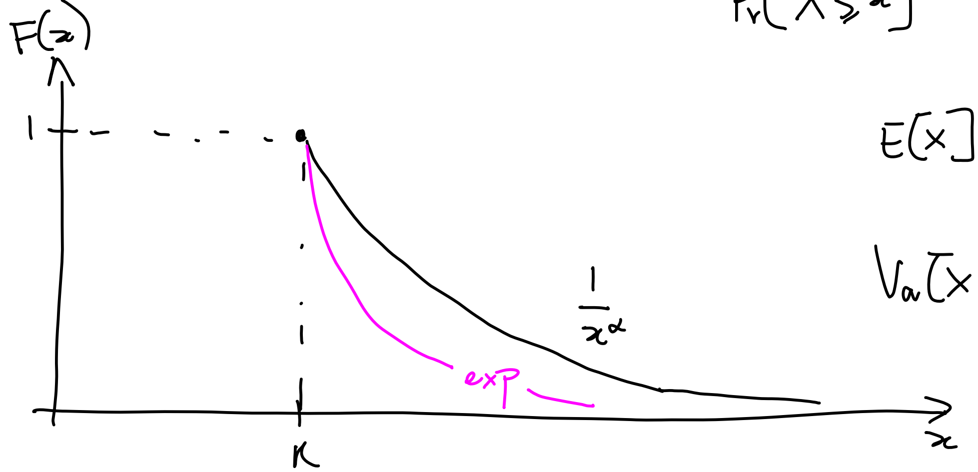
A random variable X is heavy-tailed iff $P_r[X \geq x] \sim L(x)\, x^{-\alpha}$

where $\lim_{x \to \infty} \frac{L(cx)}{L(x)} = 1 \quad \forall$ constant c.

Eg X follows a Pareto distribution $F(x) = \left(\frac{K}{x}\right)^{\alpha}$ $(0 < \alpha < 2)$. $K > 0$ $\forall_{x \geq K}$

$\shortparallel$

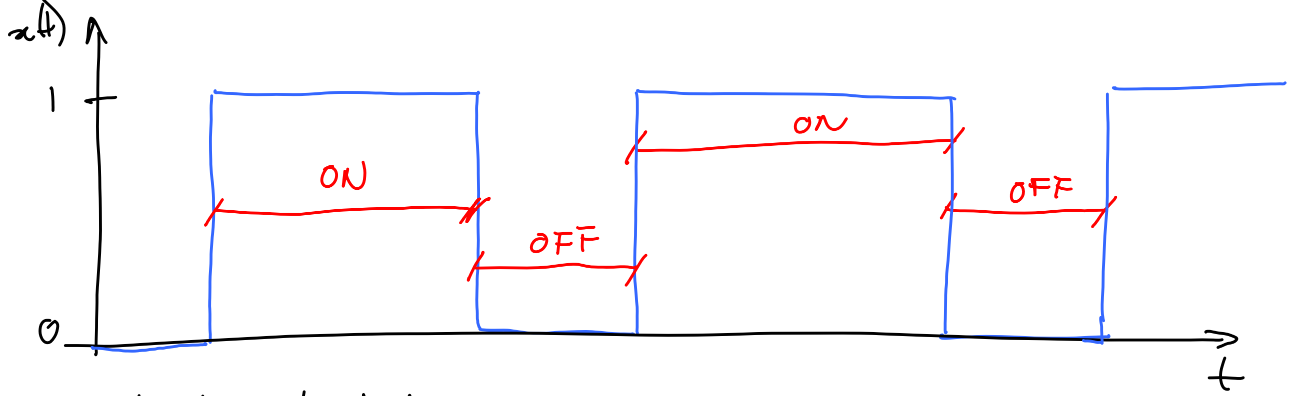
$P_r[X \geq x]$



$E[X] = \frac{\alpha}{\alpha - 1} K \quad (\alpha > 1)$ shape

$Var[X] = \infty$

ON/OFF stochastic process



Heavy tailed stochastic process

ON periods are i.i.d. heavy tailed

and/or

OFF " " " " "

$W^{(i)}(t)$ are ON/OFF heavy tailed stochastic processes

$\mu_0 =$ exp value of OFF times $\qquad \alpha_0 =$ shape OFF

$\mu_1 =$ " " ON " $\qquad \alpha_1 =$ ON

Thm

$$\mathcal{L}\lim_{T\to\infty} \mathcal{L}\lim_{M\to\infty} \frac{W_M^*(Tt) - TM\frac{\mu_1}{\mu_1+\mu_0}t}{T^H\sqrt{L_1(T)M}} = \sigma B_H(t)$$