

EECS 454
Analysis of Algorithms
2006 Spring Semester

Homework 3

Due on February 24, 2006
No late papers accepted

- Solve the following problems and exercises in CLRS:
 - 29.4-1 (page 810)
 - 29.1 (page 818)
- Write the dual of the following linear program:
$$\begin{array}{ll} \min & x_1 + x_3 \\ \text{s.t.} & x_1 + 2x_2 \leq 5 \\ & x_2 + 2x_3 = 6 \\ & x_1, x_3 \geq 0 \end{array}$$
- Statistical security of data.** The U.S. Census Bureau produces a variety of tables from its census data. Suppose that it wishes to produce a $p \times q$ table $D = \{d_{ij}\}$ of non-negative integers. Let $r(i)$ denote the sum of the matrix elements in the i th row and let $c(j)$ denote the sum of the matrix elements in the j th column. Assume that each sum $r(i)$ and $c(j)$ is strictly positive. The Census Bureau often wishes to disclose all the row and column sums along with some matrix elements (denoted by a set Y) and yet suppress the remaining elements to ensure the confidentiality of privileged information. Unless it exercises care, by disclosing the elements in Y , the Bureau might permit someone to deduce the exact value of one or more of the suppressed elements. It is possible to deduce a suppressed element d_{ij} if one value of d_{ij} is consistent with the row and column sums and the disclosed elements in Y . We say that any such suppressed element is *unprotected*. Describe a polynomial-time algorithm for identifying all the unprotected elements of the matrix and their values. (*Hint*: reduction to a maximum flow problem.)
- Phasing out capital equipment.** A shipping company wants to phase out a fleet of (homogeneous) general cargo ships over a period of p years. Its objective is to maximize its cash assets at the end of the p years by considering the possibility of prematurely selling ships and temporarily replacing them by charter ships. The company faces a known non-increasing demand for ships. Let $d(i)$ denote the demand of ships in year i . Each ship earns a revenue of r_k units in period k . At the beginning of year k , the company can sell any ship that it owns, accruing a cash inflow of s_k dollars. If the company does not own sufficiently many ships to meet its demand, it must hire additional charter ships. Let h_k denote the cost of hiring a ship for the k th year. The shipping company wants to

meet its commitments and at the same time maximize the cash assets at the end of the p th year. Formulate this problem as a minimum cost flow problem.