

## Homework 6

Due on April 21, 2006  
No late papers accepted

1. Consider a game of scissor-paper-and-stone in which the winner gets \$1 and in which your adversary is not allowed to play “stone” (you are allowed to play “stone”, however). Argue that no deterministic algorithm would achieve a positive payoff in the worst-case. Design a mixed strategy for this game, show that it is optimal, and establish your expected payoff in the worst-case.
2. In the following guessing game, you and your opponent choose a number between 0 and  $n$  and reveal it simultaneously. If both you and your adversary choose  $i$  ( $0 \leq i \leq n$ ), you win  $\$2^i$ , otherwise you do not win anything. Design a mixed strategy for this game, show that it is optimal, and establish your expected payoff.
3. Given an integer  $m \times n$  matrix  $A$  and an integer  $m$ -vector  $\mathbf{b}$ , the *0-1 integer programming problem* asks whether there is an integer  $n$ -vector  $\mathbf{x}$  with elements in the set  $\{0, 1\}$  such that  $A\mathbf{x} \leq \mathbf{b}$ . Prove that the 0-1 integer programming problem is NP-complete, but that it can be solved in polynomial time if  $A$  is unimodular. (Book hint: reduce from 3-CNF-SAT. Alternate hint: reduce from subset sum.)